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THE EFFECTIVE CONCENTRATION OF NON-TRACKING CPC's INCLUDING AN ECONOMIC ANALYSIS FOR PHOTOVOLTAICS

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ABSTRACT

Computer models are developed to use a set of hourly direct and global radiation data covering a period of one year (1), in order to analyze the behavior of three selected two-dimensional, non-tracking Compound Parabolic Collectors (CPC), operating in Albany, New York. This analysis leads to the determination of their averaged effective concentration ratio. These ratios are found to be significantly lower than the optical concentration ratios, due to the effects of the sun path and the sky condition. An economic comparison based on these results illustrates the importance of the knowledge of these parameters.

1. INTRODUCTION

Because CPC's achieve the highest possible concentration for any given acceptance angle, they are well suited for use as two-dimensional, non-tracking solar concentrators. However, as for any concentrating collector, the information developed at the theoretical (optical) concentration is incomplete. In order to judge the economic potential of such collectors, a prospective user must know their effective concentration which is defined here as the increase in energy collected by the concentrator absorber, compared to an optimized flat plate collector of equal area. This parameter is particularly important for photovoltaic electricity production, where the main reason for concentrating is lowering the cost of the energy produced.

For a two-dimensional, non-tracking concentrator, the effective concentration is a function of three geographical and meteorological factors: a) the sky condition; b) the seasonal importance of the solar span (daily variation of the zenith angle projection on a north-south vertical plane) and c) the rate of change of the declination.

Based on existing conditions in Albany, NY, the present study establishes the effective concentration ratios for three selected truncated (50%) CPC's through an hour to hour observation of their energy production during the year 1979. The three CPC's, named C_1, C_2 and C_3 have an optical concentration of 11.4, 6.3 and 3.15 respectively, and are studied with daily and/or monthly adjustments of their slope.

2. METHODS

The study proceeds in two main steps. The first is to determine the optimum monthly operating slope (s_i) of each of the CPC arrays and of the flat-plate reference collector, C_4. This is done by using monthly averaged radiation data to compute the total energy collected by the arrays as a function of their slope. The selected monthly operating slopes correspond to the maximum of these functions, and daily operating values are extrapolated from these results. Once the slopes are selected, the energy collected by the arrays' absorbers is then computed every hour during the one year period.

Key assumptions of the model: Because only low or non-concentrating arrays are studied, only optical properties are considered in the analysis. It is assumed that the absorber photovoltaic cells can be cooled so that efficiency does not vary with radiation intensity impinging on the cells. It is also assumed that this efficiency is independent of the wavelength so that it is the same for direct, diffuse and reflected components. To express the results in energy units, an efficiency of 14% is assumed.

The input data consist of hourly integrated direct solar radiation measurements, B, (Eppley Normal Incidence pyrheliometer) and global solar radiation measurements on a horizontal plane, G, (Eppley Precision Spectral pyranometer).

A single glass cover is assumed for the protection of the solar cells, (glass index: 1.53, absorption 0.0125). Transmittance

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Ed. Note: A list of metric-English-metric conversion values appears immediately preceding the Author Index.

$Rf(\theta_c)$ through this glass cover is calculated empirically from experimental curves (2):

$$\text{If } \theta_c \leq 40^{\circ},\ Rf(\theta_c) = 0.92 \qquad (1)$$

$$\text{If } \theta_c \geq 40^{\circ},\ Rf(\theta_c) = 0.92 \cos (9/5(\theta_c - 40))^2 \qquad (2)$$

θ_c is the angle between the beam radiation and a normal to the glass cover. Transmittance Rf' for non-direct sky radiation and ground reflected radiation is given an average value of 0.84 (2).

The energy E_{c4} collected by the flat reference array is the sum of three components calculated each hour from input data: direct radiation B_{c4}, non-direct sky radiation D_{c4} and ground reflected radiation R_{c4}. Assuming that diffuse and ground reflected radiation are isotropic, the three components are computed as proposed by Liu and Jordan (3). The ground albedo is assumed to be equal to 0.25 for every month but January and February, when an assumed snow cover raises its value to 0.65.

The energy, E_{ci}, collected by the CPC's is the sum of direct, B_{ci}, and non-direct sky radiation component D_{ci}, calculated each hour from input data per unit of absorber area. The troughs are theoretically infinitely long by attaching vertical mirrors at each end. According to A. Rabl (4), the direct radiation component may be written as:

$$B_{ci} = B'_{ci}\ C_i(\beta_i)\rho^{\langle n_i(\beta_i)\rangle} \times Rf(\theta_{ci}) \qquad (3)$$

where B'_{ci} is the direct component calculated for a flat plate collector having the same slope, s_i, as the considered CPC array. β_i is the angle between the orthogonal projection of the solar beam on a north-south vertical plane and a normal to the array. $C_i(\beta_i)$ is the idealized angular transmission function represented in a normalized form on Fig. 2 for a true and a truncated CPC (5). Reflectivity (ρ) of the parabolic mirrors is assumed to be 0.9 and $\langle n_i(\beta_i)\rangle$ is the average number of reflections on the side mirrors assumed to be a linearly increasing function of $|\beta_i|$. This function was extrapolated from published values of $\langle\bar{n}\rangle$, n_{max} and n_{min} within the acceptance angle of CPC's (6,7). The subscript $_i$ refers to the collector number.

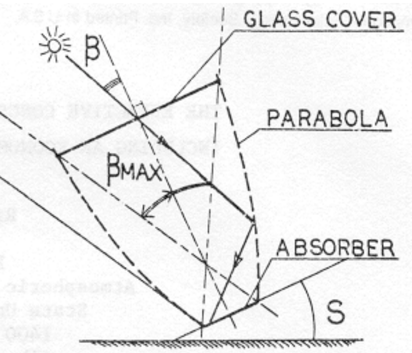

Fig. 1. Cross section of a CPC.

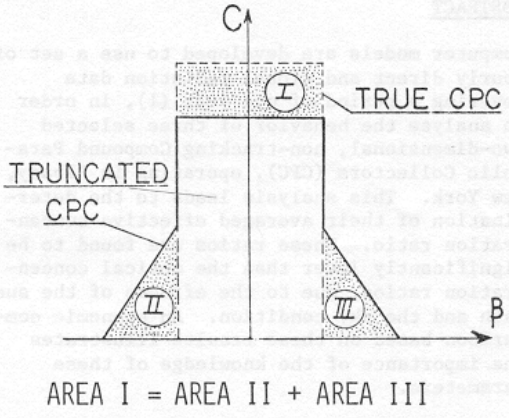

Fig. 2. Angular transmission Function of a CPC.

Referring to E.J. Guay theorem #3 (8), the diffuse power density on the absorber from isotropic radiation is the same as the diffuse power density on a horizontal plane and is independent of the geometry of the reflectors for a maximally concentrating collector. Assuming that the truncated CPC is a good approximation to such a collector and that its absorber doesn't see any portion of the ground, the non-direct component value is affected only by the global average number of reflections $\langle\bar{n}\rangle$, within the angular transmission function and may be written

$$D_{ci} = (G - B \cos \theta_h)\rho^{\langle\bar{n}\rangle} \times Rf' \qquad (4)$$

where θ_h is the zenith angle.

3. RESULTS

The process of selecting an optimum slope is illustrated in Figures 3 and 4 where the energy collected by the selected arrays is plotted against slope for each collector for June 1979. The precision needed in selecting an optimal slope increases with the degree of concentration. This optimal slope is smaller than the average noon zenith angle for the month ($\phi - \bar{\delta}$). This is due to the effect of the solar span. Following the spring equinox and preceding the fall one, the sun lies above an east-west plane which lies on the line defined by the noon altitude of the sun. The extent of the span is then equal to ($\phi - \delta + \pi/2$) and has a maximum effect at the summer solstice. In winter this effect is reversed because the sun lies below such a plane, although the extent of the solar span is smaller and equal to ($\pi/2 - \phi + \delta$). As a consequence,

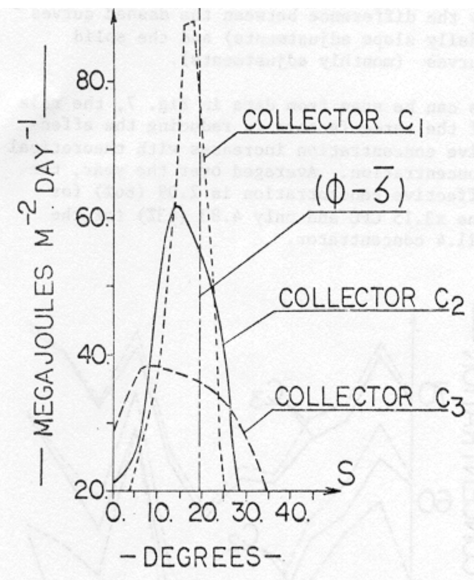

Fig. 3. Energy per m^2 of absorber collected by the concentrating arrays as a function of their slope in June 1979.

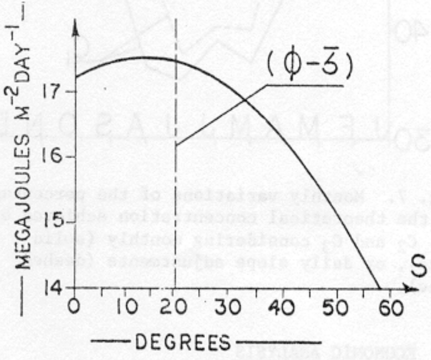

Fig. 4. Energy per m^2 received by the reference flat-plate collector as a function of its slope in June 1979.

the optimum winter slopes are found to be greater than $(\phi - \bar{\delta})$. These results are summarized in Fig. 5 for C_1, C_2 and C_3, where the monthly values of deviation of the optimum slope from $(\phi - \bar{\delta})$ are plotted, the shaded parts corresponding to a negative deviation. Also represented on these figures is the percentage of collected energy that would be lost if the slope were set at $(\phi - \bar{\delta})$ instead of the computed optimum value.

The solar span effect alters the energy production of the CPC arrays, increasingly with the degree of concentration. Around the summer solstice, and to a lesser extent, around the winter one, the sun lies outside the acceptance angle of a CPC for a good part of the day, (morning and afternoon), during which time a flat plate collector still receives a substantial amount of beam energy which decreases the value of the CPC's effective concentration. For the highest concentrating array, this effect is coupled with the consequences of the declination rate of change when the array is adjusted only once a month. This combined effect is maximum around the equinox. An example of this effect is shown in Fig. 6 representing the October 6th energy production, (E_{ci} x cell efficiency), of the three CPC's. At peak production time, the sun is outside the maximum part of the angular acceptance of C_1.

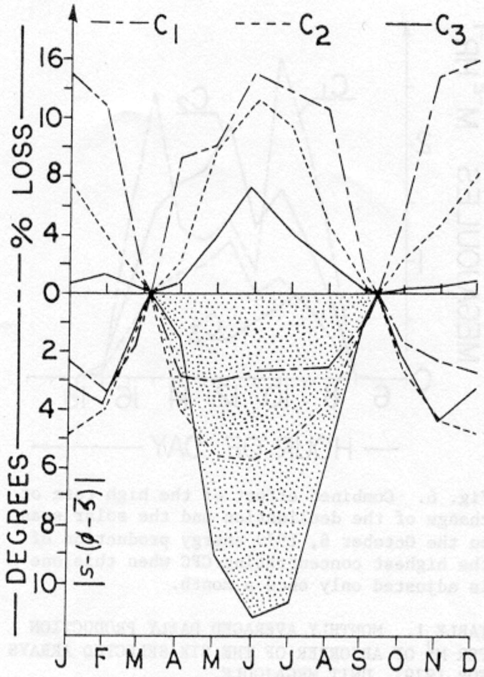

Fig. 5. Monthly variations of the optimum slope deviation from $(\phi - \bar{\delta})$ in 1979 for the concentrating arrays and percentage of energy lost if the slopes are set at this value instead of the optimum value.

The 1979 production achieved by the three CPC's and the reference flat plate collector is summarized in Table 1. Using these results, it is then possible to compute the monthly averaged effective concentration ratios by calculating the energy production ratios between the CPC's and the reference collector. These values are plotted in Fig. 7 as the percentage of theoretical concentration achieved by the three arrays

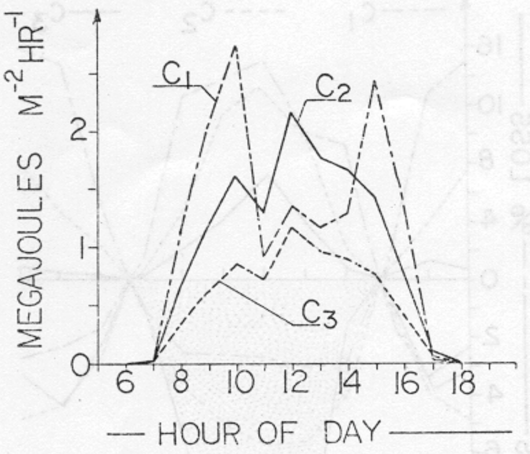

Fig. 6. Combined effect of the high rate of change of the declination and the solar span on the October 6, 1979 energy production of the highest concentrating CPC when this one is adjusted only once a month.

TABLE 1. MONTHLY AVERAGED DAILY PRODUCTION PER M^2 OF ABSORBER OF THE SIX SELECTED ARRAYS FOR 1979: UNIT MEGAJOULE.

Month	C_1	C_1^*	C_2	C_2^*	C_3	C_3^*	C_4
J	5.1	5.3	3.4	3.5	2.0	2.0	0.9
F	11.8	13.8	8.2	8.5	4.6	4.7	2.0
M	7.4	10.0	5.7	5.9	3.2	3.3	1.5
A	9.5	12.1	7.1	7.4	4.2	4.3	2.0
M	9.2	10.5	6.9	7.2	4.4	4.4	2.3
J	12.0	12.5	8.5	8.8	5.2	5.5	2.7
J	10.0	10.8	7.3	7.6	4.6	4.7	2.5
A	8.7	10.7	6.7	7.1	4.2	4.2	2.1
S	11.7	17.1	9.4	9.9	5.2	5.3	2.3
O	6.0	7.0	4.3	4.5	2.5	2.6	1.2
N	5.1	6.0	3.7	3.8	2.2	2.2	1.0
D	6.3	6.3	4.1	4.1	2.3	2.3	1.0

Note: The superscript* indicates daily slope adjustments for C_1, C_2 and C_3.

with daily and monthly slope adjustments. The importance of the sky condition is illustrated by the local peaks in February and September and, to an extent in June and December, which were relatively clear months in 1979. In October and January, both of which were cloudy, the percentage values are relatively low. The low summer ratios in Fig. 7 are due to the solar span effect. The only possible improvement would require some degree of sun tracking. The consequences of the declination rate of change on the effective concentration ratios is illustrated by the difference between the dashed curves (daily slope adjustments) and the solid curves (monthly adjustments).

As can be seen from data in Fig. 7, the role of the three factors in reducing the effective concentration increases with theoretical concentration. Averaged over the year, the effective concentration is 2.09 (66%) for the x3.15 CPC and only 4.83 (43%) for the x11.4 concentrator.

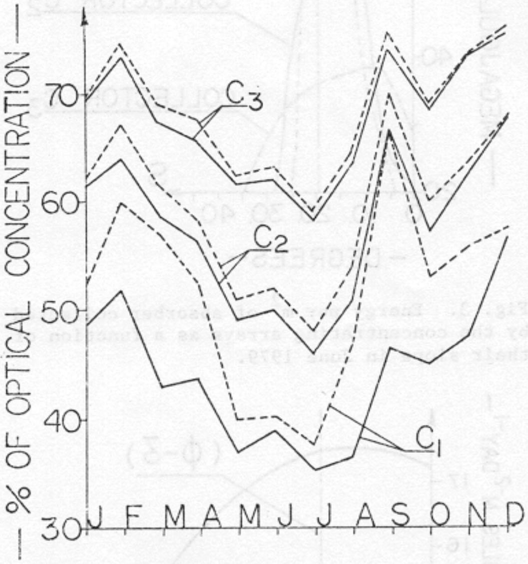

Fig. 7. Monthly variations of the percentage of the theoretical concentration achieved by C_1, C_2 and C_3 considering monthly (solid line), or daily slope adjustments (dashed line).

4. ECONOMIC ANALYSIS

The effective concentration is an essential piece of information for those who purchase concentrating photovoltaic collectors by the conventional measure of price per peak Watt. The following economic comparison which uses the results obtained in this study is based on the calculation of the ratio between the total cost of each array and the total energy produced during its lifetime as a function of the cost of photovoltaic solar cells. Computations are made using constant dollars and the arrays are financed without a loan.

The total cost of each array is computed from: (a) cost of supporting structure using \$40 per m^2; (b) cost of parabolic mirrors of \$25 per m^2; (c) maintenance and operation cost per year of 1.5% of total cost for arrays adjusted monthly and 2% for arrays adjusted daily; (d) cost of silicon solar cells (@ \$1300 per m^2 in 1980). The lifetime of all the collectors is assumed to be 15 years and their total energy production is assumed to be 15 times their 1979 production.

The results are given in Fig. 8 where cost per kWh produced is plotted versus solar cell cost.

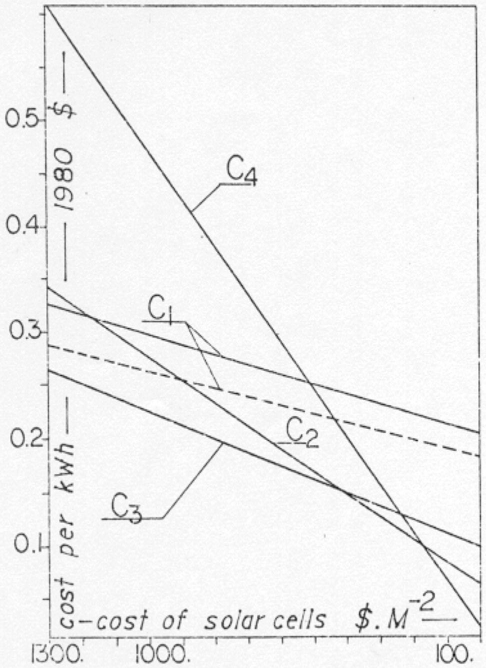

Fig. 8. Variation of the cost of the energy produced with the cost of solicon solar cells. The dashed line corresponds to a daily adjustment of C_1's slope.

The "sensitivity" of a collector to the cells' price is expressed by its slope, and as might be expected, this slope is primarily a function of the degree of concentration.

One point needs to be underlined. The x11.4 CPC is already uneconomical when compared to a x6.3 CPC. This conclusion would have been different, had the comparison been based only on the price per peak Watt. All the selected CPC appear to be more economical now than the flat plate collector under the assumptions of this analysis but this changes as the cost of the cells gets below \$450/m^2 for C_1, \$275 for C_2 and \$220 for C_3. These figures derived from the calculation of effective concentrations are much larger than one would obtain using only the information developed at the theoretical concentration, because they include the meteorological and regional characteristics of the operating area.

5. CONCLUSION

The most important conclusion is that the effective concentration ratios are significantly reduced from the theoretical values. The seasonal effect of the solar span on the effective concentration is predictable, but the effect of the sky condition is not. The magnitude of the solar span effect would generally be the largest under normal weather conditions. The effect of the declination change is important only near the time of the equinoxes for high concentrating arrays.

For any given price of silicon cells there is an economically optimum optical concentration ratio, which is lower than that which would be calculated from the conventional procedure using price per peak Watt.

The effective concentration as used here, averages all of the effects characterizing a region. The user must keep in mind that for a specific use such as maximizing the output for high insolation days only, the optical concentration may prove to be a more useful information.

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