CALCULATING SOLAR RADIATION RECEIVED
BY TUBULAR SOLAR ENERGY COLLECTORS

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ABSTRACT

Cylindrical (tubular) absorbers installed inside evacuated tubes represent an increasingly common design for low temperature solar collectors (e.g., the SunFamily^{TM} design). However, whereas much work has been done on the subject of solar radiation received by flat plate collectors, little has been done for collectors of tubular design.

It is important to estimate the irradiance impinging on a collector to: (1) evaluate its efficiency and (2) be able to predict its performance in a given location. Applying flat plate irradiance calculations without change to tubular collector arrays would result in sizable errors.

In this paper we present and discuss the main assumptions of an algorithm developed by the authors to estimate irradiance impinging on tubular arrays, including specific collector/array design parameters, treatment of direct and anisotropic diffuse radiation, treatment of shading from one tube to another and treatment of ground and support-reflected radiation. Key examples are provided to illustrate the difference of energy collected between flat plate and tubular collectors.

INTRODUCTION

Before addressing the modeling issue per se, it is useful to first point out some notable distinctions between a tube and a flat plate collector. The two most important ones are:

(1) Point source radiation impinging on a tube is subject to the cosine law in only one dimension: along the tube's axis. In the other dimensions (a plane perpendicular to the tube's axis) radiation is always normal incident.

(2) Tubes receive radiation from all directions: A free standing array of tube (a set of parallel tubes) receives radiation from the sun, the sky, and the ground (or other structures) on the front of the array, as well as on the back of the array.

Other important considerations include shading effects from one tube to another and the fact that solar energy that penetrates between tubes and may partially by absorbed by the array after reflection.

METHODS

Defining An Array of Tubular Collectors

We start by describing the design parameters and assumptions which are relevant to the determination of insolation received by an array of tubes.

The first step in this process is to define the considered absorber area. For a single tube this is defined here as the projection of the tube in a plane parallel to its axis. For an array composed of several tubes, the collector area consists of the sum of the areas of each individual tubes.

The second step is to define an *"array of tubes"* as opposed to a single tubular collector. An array of tubes is defined as the rectangle formed by a set of parallel tubes. In this paper, we consider two most common *array types* illustrated in Fig. 1. Type 1 arrays are composed of tubes tilted along with the array, and type 2 arrays are composed of horizontally mounted tubes. In addition, we consider two possible *mounting configurations* shown in Fig. 1. Configuration A corresponds to a free standing array (e.g., tubes mounted in a parapet configuration) while configuration B corresponds to a standard tilted roof mount, with a supporting plane parallel to the tube array.

Given these definitions the design parameters relevant to insolation calculations include:

- The slope and orientation of the array, respectively S and γ
- The diameter of the tube's absorber, D
- The diameter of the tube's glass cover, D_p
- The spacing between each parallel tube, D_r
- The number of parallel tubes, n
- The albedo of the surrounding ground, a
- In the case of configuration B, the distance between the support and the array, L
- Also in the case of configuration B, the albedo of the support (e.g., the roof), a_R
- The transmissivity of the tube's glass cover at normal incidence t_o, and its relative variation with incidence $f(incidence)$

Many of these parameters are illustrated in Fig. 2.

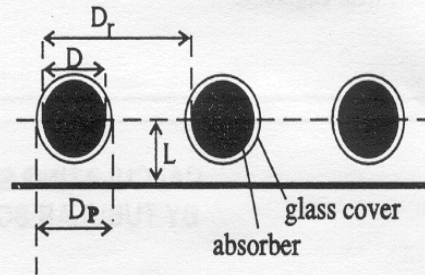We finally assume that tubes are true cylinders and do not account for any tube's end effects.

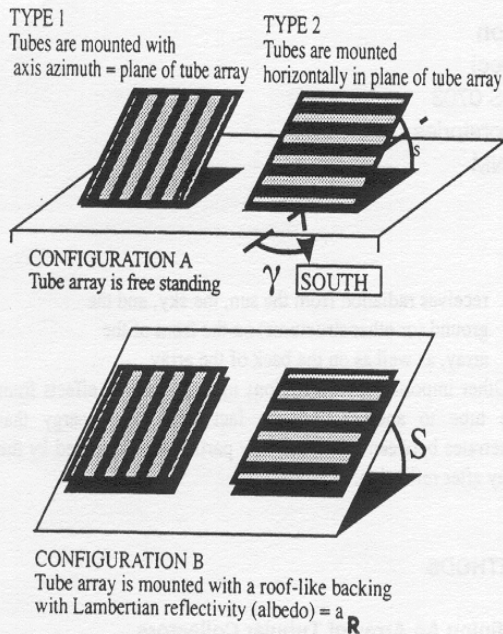**Figure 2:** Description of key array design parameters

Figure 1: Array configurations and types

Treatment of Direct Irradiance

Because of its symmetry of rotation, a tube will intercept exactly as much parallel beam radiation as an ideal one-axis tracking flat plate collector whose axis coincides with the tube and whose area equals that of the tube as defined above. The problem, therefore, reduces to calculating the solar incidence angle θ_{tube} on a one-axis tracking collector of arbitrary axis' slope and orientation. This angle is obtained from:

$$\cos \theta_{tube} = [\cos \theta_1] / [\cos \alpha]$$

where θ_1 is the solar incidence angle on a plane with the same slope and orientation as the tube's axis (see Fig. 3) -- *note that incidence on an arbitrary plane may be determined using well known formulations (e.g., Iqbal, 1983)* -- and where α, the projection of the solar incidence angle on a plane normal to the tube's axis, is given by:

$$\alpha = \mathrm{atan2}^{\clubsuit} [\cos \theta_1, \cos \theta_a]$$

where θ_a is the angle of incidence on a vertical plane containing the tube's axis and facing east of the tube (see Fig.3).

The direct irradiance impinging on the tube, B_c, is equal to B multiplied by $\cos \theta_{tube}$, where B is the normal direct irradiance.
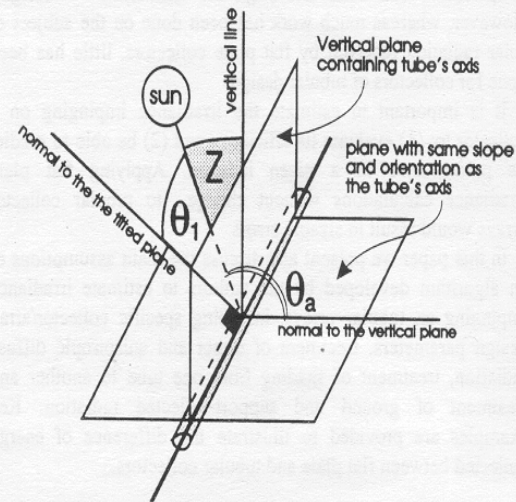

Figure 3: Key incidence angles

Treatment of Diffuse Irradiance

We make use of the sky model developed by Perez et al. (1990). This model describes the sky as an isotropic background

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$^{\clubsuit}$ the function atan2(x,y) returns the arc tangent angle between 0 and 2π, defined by its two Cartesian coordinates x and y

against which are superimposed a circumsolar region (reduced to a point source) and an horizon brightening/darkening zone.

For the present application, we make the following assumption: the circumsolar region is used as described in the model, however the rest of the sky is considered to be isotropic in the region faced by the tube array -- this allows us to fully incorporate the horizon band effect while reducing the problem to the treatment of a diffuse point source and a diffuse isotropic source. This assumption is reasonable and safe, because the horizon band is an approximation that is valid only for flat plate calculations and that should not be used for complex collector geometries (Perez et al., 1990).

Circumsolar Diffuse. The circumsolar portion of diffuse irradiance, Dc_{circum}, impinging on a tube is equal to

$$Dc_{circum} = Dh * f_1 * \cos \theta_{tube} / [Max^\bullet (0.087, \cos Z)]$$

where Dh is the horizontal diffuse irradiance, f_1 is the circumsolar enhancement parameter obtained from Perez et al. (1990), and Z is the solar zenith angle.

Isotropic Diffuse. The amount of anisotropic irradiance received by a cylinder may be formally calculated as follows. Given R, the uniform radiance from the hemisphere facing the tube and the angles Φ_1 and Φ_2 describing this hemisphere (see Fig. 3), the isotropic irradiance, D_{iso}, received by the tube may be formulated as

$$D_{iso} = \int_{\Phi_2 = 0}^{\Phi_2 = \pi} \int_{\Phi_1 = -\pi/2}^{\Phi_1 = \pi/2} R \cos^2 \Phi_1 \, d\Phi_1 \, d\Phi_2$$
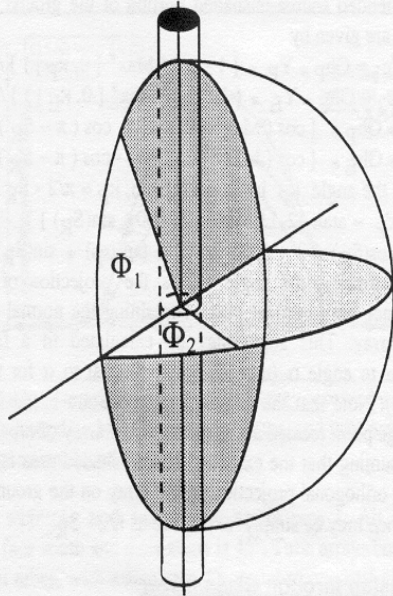
Figure 4: Illustration of angles used for the calculation of isotropic radiation received by a tube

which leads to $D_{iso} = \pi^2/2 \, R$, that is $\pi/2$ times more than a flat plate facing the same hemisphere. Hence, based on the

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$\bullet$ the function Max returns the largest of two expressions separated by a comma. (e.g., if x > y, then Max(x,y) = x

assumption formulated above, the isotropic diffuse radiation, Dc_{iso}, received by the tube is

$$Dc_{iso} = Dh * [(1-f_1) (1 + \cos S) / 2 + f_2 \sin S] * \pi/2,$$

where S is the slope of the tube array, and where f_2 is the horizon enhancement coefficient from Perez et al. (1990).

In the case of configuration A (free standing) , the tube array also receives diffuse isotropic radiation on its back side $D'c_{iso}$, given by

$$D'c_{iso} = Dh * [(1-f_1) (1 - \cos S) / 2 + f_2 \sin S] * \pi/2,$$

It follows that the total unobstructed sky diffuse radiation received by the array, Dc is

$$Dc = Dc_{circum} + Dc_{iso} + D'c_{iso}$$

Treatment of Reflected irradiance

Radiation reflected from the ground. We make the standard -- and generally considered most accurate (Ineichen et al.., 1990) -- assumption that ground reflected radiation is isotropic. Therefore, based on the discussion above, and expanding on the standard relation from Liu & Jordan (1963) the radiation impinging on the front of the array after reflection on the ground, Rc, is obtained from

$$Rc = Gh * a * (1 - \cos S) * \pi/4$$

where Gh is the global irradiance on the horizontal, and where a is the albedo of the ground.

For configuration A (free standing) , the tube array also receives ground-reflected radiation on its back side $R'c$, given by

$$R'c = Gh * a * (1 + \cos S) * \pi/4.$$

Radiation reflected from the roof or other support underneath the array (configuration B only): We make the assumption that the roof, as the ground, is a Lambertian (isotropic) reflector. Under this assumption the radiation, RRc, received by the array after reflection on the roof may be obtained from

$$RRc = G_R * a_R * \pi/4$$

where G_R is the global irradiance impinging on the roof and a_R is its the albedo.

Shading effects

There are two types of shading effects which tend to reduce the amount of energy received by an array of tubes. These two effects are: (1) direct obstruction of incoming radiation, and (2) indirect obstruction via shadowing of reflecting surfaces. All shading effect calculations described below assume that tubes are ideal absorbers and do not reflect radiation.

Direct Obstruction: Tubes receive radiation from two types of sources, (1) point sources -- the sun and circumsolar diffuse -- and (2) extended sources -- the rest of the sky and the ground. Shading effects are calculated differently for each.

Point source shading: The approach we propose is based on the following observation: Since an ensemble of parallel collectors cannot receive more radiation than is impinging on the area formed by the ensemble of the

collectors, the shadowing coefficient, X_P, can simply be estimated as

$$X_P = 1 - Max^{\blacktriangledown} [\, 0, (D * \cos \theta_{tube} - D_r * \cos \theta)\, / (D * \cos \theta_{tube})\,] * (n-1)/n$$

where D and D_r are the tube diameter and the inter-tube distance respectively, the angle θ is the solar angle of incidence on the plane of the tube array, and n is the number of tubes. The direct and circumsolar diffuse irradiance components, Bc and Dc_{circum} should be multiplied by X_P to account for the obstruction of point source radiation.

Extended source shading: Because the angular extent of sky and ground/support-reflected extended source radiation impinging on both sides of the collector are about equal, and because of symmetries in the isotropic shading patterns for an array of tubes, it is reasonable to assume that one unique shading factor will apply to the entire isotropic radiation field of view (composed of isotropic sky and isotropic reflected radiation). The isotropic shading coefficient, X_E is equal to one minus the lost portion of field of view due to adjacent tubes, adjusted for the end tubes, which experience loss of field of view on one side only. This is given by

$$X_E = 1 - 2/\pi * asin (D / 2D_r) * (n-1)/n$$

The radiation components Dc_{iso}, $D'c_{iso}$, Rc, R'c and/or RRc, as appropriate should be multiplied by the isotropic field shading factor X_E

Indirect Obstruction: We consider the two primary indirect shading effects: (1) the shadowing cast by the tube array on the roof or support for configuration B, and (2) the shadowing cast by the array on the ground on the back of the array for configuration A. Shadowing effects on the ground in front of the array (that may occur for non-south facing array or early morning in summer) is ignored as is standard practice for flat plate calculations. In each case the problem is treated separately for point source and diffuse effects.

Configuration B arrays: The first step is to find the amount of unobstructed radiation G_R received by the roof underneath the array and to decompose this quantity into its point source and extended source components. based on the above assumption, the point source component G_{RP} is obtained from:

$$G_{RP} = \{B + Dh * f_1 / [Max^{\blacktriangledown} (0.087, \cos Z)]\,\} * Max^{\blacktriangledown} (0, \cos \theta),$$

while the extended source component is equal to

$$G_{RE} = Dh * [\, (1-f_1)(1 + \cos S_R)/2 + f_2 \sin S + Gh * a * (1 - \cos S_R)$$

where S_R is the slope of the tube array.

The second step is to determine how much point source and extended source radiation passes through the tube array and reaches the surface below. As an extension of the reasoning developed for point source direct shading, the fractional amount of point source radiation, y_P, passing through the array composed of an infinite number of tubes and reaching the surface below may be simply expressed by:

$$y_P = 1 - Max^{\blacktriangledown} [\, 0, (D_r * \cos \theta - D * \cos \theta_{tube}) / (D_r * \cos \theta)\,]$$

Adjusting for the number of tubes in the array, and adding a lower bound consisting of the indirect shading effect of a single tube, the fractional amount of point source radiation passing through the array, Y_P, is

$$Y_P = Min^{\bullet} [\, (1 - 2 * atan(D/2L) / \pi)\,, \; yb_B * (n-1)/n]$$

Similarly, the fractional amount of extend source (isotropic) radiation passing through the array, Y_E is approximated by

$$Y_E = Min^{\bullet} [\, (1 - 2 * atan(D/2L) / \pi)\,, \; (1 - D/D_r)*(n-1)/n]$$

The component RRc (radiation received by the array after reflection on roof) is expressed as follows after correction for indirect shading

$$RRc = (G_{RP} * Y_P + G_{RE} * Y_E) * a_R * \pi/4$$

Configuration A arrays: The problem is quite similar here. However, the impact of the array on the back reflective surface (i.e., the ground here), is limited to the size the shadow cast by the array which is dependent on solar geometry.

The first step, as above, is to differentiate between point source, Gh_P, and extended source, Gh_E, global radiation impinging the ground. These are given by

$$Gh_P = B * \cos Z + Dh * f1, \quad \text{and} \quad Gh_E = Dh * (1-f1)$$

The second step is to individually compute the amount of reflected radiation on the back of the array from (1) the point source shaded portion of the ground, $R'c_1$, (2) the extended source-shaded portion of the ground, $R'c_2$, (3) the point source-unshaded portion of the ground, $R'c_3$, and (4) the extended source-unshaded portion of the ground, $R'c_4$. These are given by

$$R'c_1 = Gh_P * Y_P * \{1 - \cos (Max^{\blacktriangledown} [\, 0, \kappa_P]\,)\} / 2$$
$$R'c_2 = Gh_E * Y_E * \{1 - \cos (Max^{\blacktriangledown} [\, 0, \kappa_E]\,)\} / 2$$
$$R'c_3 = Gh_P * \{\cos (Max^{\blacktriangledown} [\, 0, \kappa_P]\,) - \cos (\pi - S_R)\} / 2$$
$$R'c_4 = Gh_E * \{\cos (Max^{\blacktriangledown} [\, 0, \kappa_E]\,) - \cos (\pi - S_R)\} / 2$$

where the angle κ_P is obtained from, $\kappa_P = \pi/2 - S_R + \kappa'_P$, with $\kappa'_P = atan \{2*L''/Max^{\blacktriangledown} [0.00001, \sin(S_R)]\,\}$, where $L'' = \cos(S_R) / 2 + L'$, with $L' = \tan(\alpha_P) * \sin(S_R)$, and where, finally, the angle α_P is the projection of solar incidence on a vertical plane containing the normal to the tube array. This angle may be calculated in a fashion similar to angle α (note that α_P is equal to α for type 2 arrays). Note that the above calculations are based from a vantage point located at the center of the array of tube.

Assuming that the extended source shaded area is equal to the orthogonal projection of the array on the ground, the angle κ_E may be simply expressed as $\pi/2 - S_R$

Transmission through Glass Cover

The first step in this process is to select a glass cover transmission function (e.g., fitted from the Window program, 1992). These functions typically include two components (1) a

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$^{\bullet}$ the function Min returns the smallest of two expressions separated by a comma. (e.g., if x > y, then Min(x,y) = y

$^{\blacktriangledown}$ the function Max returns the largest of two expressions separated by a comma. (e.g., if x > y, then Max(x,y) = x

normal incidence transmission coefficient, t_o, and a function, f, dependent on incidence angle β. Transmission through the glass cover is expressed as:

$$t(\beta) = t_o * f(\beta).$$

The second step is to calculate an "optimal incidence" transmission for the considered tube. Indeed, because of its cylindrical shape, true normal incidence through the glass cover will never be achieved. Under optimal conditions (i.e., when the point source is in a plane perpendicular to the tube's axis), a portion of the radiation will reach the absorber after transmission at high incidence through the edges of the glass cover. This optimal transmission, t_{OPT}, may simply be calculated via integration of $t(\beta)$, and is a function of the tube and glass cover diameters.

It follows that, for point source radiation, the glass transmissivity, t_P, function is given by:

$$t_P = t_{OPT} * f(\theta_{tube})$$

For extended source radiation, the considered glass transmissivity, t_E, is approximated to a point source with an average incidence angle of 45°.

RESULTS

The new algorithm is illustrated by simulating the amount of radiation received by a flat plate collector and two tube arrays -- of configuration A and B respectively -- during winter, equinox and summer clear days. The input data for this example consists of global and direct irradiance recorded in Albany, New York (Perez et al., 1994).
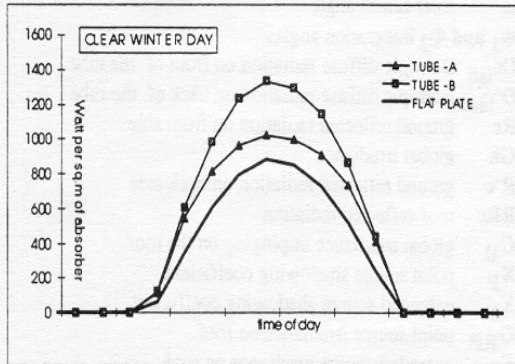

Figure 4: Comparison between the energy received on a clear winter day by a flat plate collector, and tube arrays of configuration A and B

In this example, both tube arrays are of type 1. All considered arrays face south and their slope is $45°$. Tube arrays consist of 10 parallel tubes, with a diameter, D, equal to one half of the inter-tube distance D_r. The diameter of the tubes' glass covers, D_p, is 20% larger than D. For the configuration B array, the distance between the roof and the tubes, L, is taken equal to D_r. The albedo of the roof, a_R, is assumed to be 0.7, and that of the ground, a, is set at 0.2. Finally, the normal incidence glass transmissivity, t, is set at 100%. The variation of transmissivity with incidence angle corresponds to that of a generic single pane glass cover.

Figures 4, 5 and 6 correspond to winter, equinox and summer operation. Their most striking feature is the increase in solar radiation collected by the tube over that collected by the flat plate collector. It is also interesting to remark that the B-
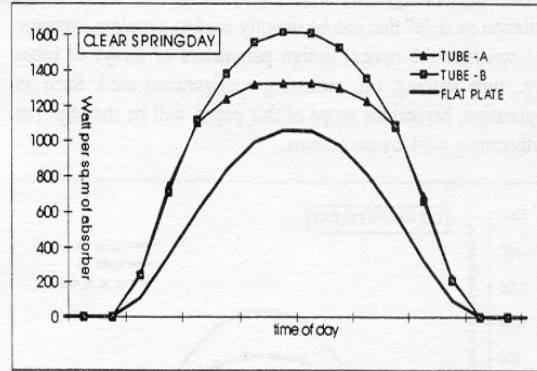

Figure 5: Same as Fig. 4, but clear spring day

configured array receives more radiation than the A-configured array because of the selected highly reflecting roof (0.7 albedo) -- note that without accounting for indirect shading on the roof, the gain of array B over array A would have been estimated to be 500 W/m^{-2} higher at noon time than reported on Fig. 4-6. However, in early morning and late afternoon summer, the A-array is ahead because it receives beam radiation from the back, while the B-array is shaded by the roof. It is finally important to remark in Fig 8, that when considering energy collected per unit
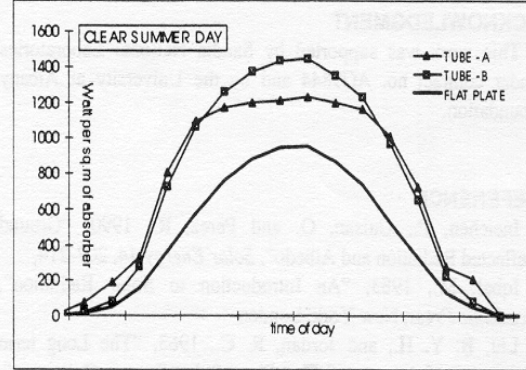

Figure 6: Same as Fig. 4, but clear summer day

of array (roof) area, as opposed to absorber area, the flat plate collector receives the largest amount of energy -- this is to be expected because of the non-energy collecting space between the tubes.

CONCLUSIONS

We presented a straightforward methodology to calculate short time step (e.g., hourly) insolation impinging on tubular collectors arrays of arbitrary configuration. This methodology includes simple but effective approaches for the calculation of direct, diffuse, reflected irradiance and direct shading effects (i.e., the obstruction created by tubes onto one another). The proposed methodology also accounts for indirect shading effects

(i.e., the obstruction created by tubes onto adjacent reflecting surfaces) which tend to make the simulation process heavier, but which cannot be ignored, especially if the tube array is mounted on a high albedo support..

The methodology has been incorporated into a "front-end radiation module" that can be directly used to simulate, compare and optimize the optical design parameters of arrays of tubes (e.g., tube spacing, tilt, mounting configuration etc.). Such an application, beyond the scope of this paper, will be the object of forthcoming work by the authors.
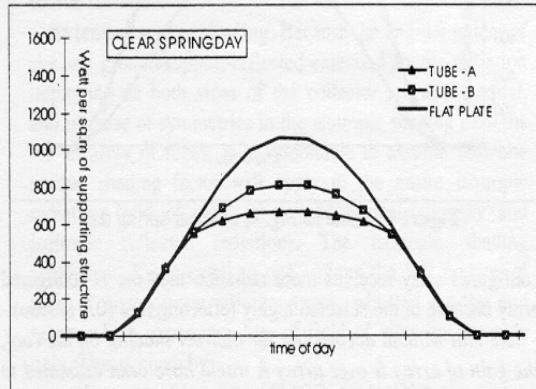
Figure 7: Comparison between the energy received *per unit area of supporting structure,* on a clear spring day by a flat plate collector, and tube arrays of configuration A and B

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NOMENCLATURE

S	slope the array
γ	orientation of the array
D	diameter of the tube's absorber
D_p	diameter of the tube's glass cover
D_r	spacing between each parallel tube
n	number of parallel tubes
a	albedo of the surrounding ground
L	in the case of configuration B, distance between the support and the array,
a_R	also in the case of configuration B, albedo of the support (e.g., the roof),
t_o	transmissivity of the tube's glass cover at normal incidence,
f	relative variation of t_o with incidence angle -- f(incidence)
t	relative glass transmissivity
t_{OPT}	average glass cover transmissivity for normal incidence on the tube
t_p	glass cover transmissivity fro point source radiation
θ_{tube}	effective solar incidence angle on the tube
θ_1	solar incidence angle on a plane with the same slope and orientation as the tube's axis
α	projection of the solar incidence angle on a plane normal to the tube's axis
θ_a	angle of incidence on a vertical plane containing tube's axis and facing east of the tube
B	normal direct irradiance
Dh	horizontal diffuse irradiance
Dc_{circum}	circumsolar portion of diffuse irradiance
f_1	Perez model circumsolar enhancement parameter
f_2	Perez model horizon enhancement parameter
Z	solar zenith angle
Φ_1 and Φ_2	integration angles
Dc_{iso}	isotropic diffuse radiation on front of the tube
$D'c_{iso}$	isotropic diffuse radiation on back of the tube
Rc	ground reflected radiation on front side
Gh	global irradiance
$R'c$	ground reflected radiation on back side
RRc	roof-reflected radiation
G_R	global irradiance impinging on the roof
X_P	point source shadowing coefficient
X_E	extended source shadowing coefficient
G_{RP}	point source irradiance on roof
G_{RE}	extended source irradiance on roof
y_P	fractional amount of point source radiation on roof for infinite number of tubes
Y_P	fractional amount of point source radiation on roof for finite number of tubes
Y_E	fractional amount of extend source radiation on roof
Gh_p	point source radiation on ground
Gh_E	extended source radiation on ground
$R'c_i$	i = 1, 2, 3, 4 point source shaded, extended source shaded, point and extended unshaded ground reflected radiation
$\kappa_p, \kappa_E, \kappa'_p, \alpha_p, L'', L'$	intermediate angular quantities for estimating shadow cast by array on the ground